BLOOD EXCHANGE IN REPLACEMENT TRANSFUSIONS

I. THEORETIC CONSIDERATIONS

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EXCHANGE transfusions have been accepted as the therapy of choice in severe cases of erythroblastosis fetalis. Bessis and Bernard1 and others2–5 have reported the value of replacement transfusions in patients with acute leukemia, anemia, toxemias, anuria, etc. The technics employed in these transfusions in children and adults have been adequately described6–8; the advantages of any one particular procedure over the others are relatively unimportant.

The indications for exchange transfusions are fairly evident: (1) in infants with erythroblastosis fetalis, the removal of as many Rh positive cells as possible and their replacement with Rh negative cells; (2) increasing the total circulating red cell mass to a normal level in anemic individuals without increasing the total blood volume; (3) removal by replacement transfusion of large amounts of toxic products retained in the plasma of patients with anuria due to various causes; (4) inducing remissions in acute leukemia. Our results in 10 cases of acute leukemia treated with replacement transfusions have been poor; the value of this method of treatment is still to be established.

In planning an exchange transfusion in erythroblastosis fetalis it is desirable to obtain the maximum exchange of red cells using the minimum amount of blood and, if anemia is present, to correct it. Where noxious agents circulating in the plasma are to be removed, a maximum exchange of plasma is desirable with the red cell volume kept at an optimum level. The replacement should be done in the least traumatic way, which implies that the original blood volume should be disturbed as little as possible during the course of the procedure, unless the initial blood volume is pathologically high or low. In these cases, adequate measures should be taken at the beginning or end of the exchange transfusion to correct the abnormality.

Two distinct methods of exchange transfusions are possible:

1. Discrete, intermittent or discontinuous exchange in which equal volumes of blood are administered and withdrawn alternately (or vice versa). Numerous variations of this technic are possible: the volumes administered may be smaller or larger than those withdrawn; the initial volume withdrawn or administered may be larger than successive increments, thus reducing or increasing the circulating blood volume for the duration of the exchange transfusion; at the end of the transfusion the total circulating blood volume may or may not be brought back to the initial level.

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2. Continuous exchange in which blood is injected and withdrawn simultaneously and at the same rate. This method is probably less shocking to the cardiovascular system than the discrete. Variations of this method can be achieved by removing or adding a known volume prior to the start of the continuous exchange, and by a final readjustment of the circulating blood volume if so desired.

The expected results of any exchange can be calculated. This has previously been indicated by other investigators. Wallerstein and Brodie computed the expected results in substitution transfusions involving the exchange of one homogeneous solution with another. In dealing with a nonhomogeneous medium such as blood, the relative concentration of different constituents may vary in two different individuals; therefore, changes in the final concentration of the respective constituents, after the blood is mixed, do not follow identical patterns. If equal volumes of blood A and B are mixed, the resultant mixture will contain 50 per cent of blood A and 50 per cent of blood B. However, it would be inaccurate to assume that 50 per cent of the erythrocytes of the mixture will be A cells unless the packed red cell volumes of A and B are equal. Should the hematocrit of A be 20 per cent and that of B be 40 per cent, then only 33.3 per cent of the mixed cells will be A cells. Formulae previously published for replacement transfusions are valid only if the ratio of donor hematocrit to patient hematocrit is unity. Mollison was the first to point out the value of using fairly concentrated cell suspensions as the medium of exchange.

Expected values for the dilutions of the original cells and plasma during and at the end of any type of exchange transfusion can be calculated. The data necessary for such calculations are: (1) total blood volume of the patient at the beginning of exchange, (2) patient’s venous hematocrit, (3) increments of blood withdrawn and injected, (4) total volume of the blood used in exchange, (5) hematocrit of the donor’s blood.

**Discrete Exchange Transfusions**

Let $V_0 =$ initial blood volume of the patient,

$v =$ volume of blood withdrawn (and subsequently administered) during any one discrete exchange,

$T =$ total volume withdrawn (and administered) during $n$ exchanges,

$E =$ volume withdrawn (and administered) during $n$ exchanges, expressed in multiples of $V_0$.

After each withdrawal and administration the fraction of the original whole blood of the patient remaining in circulation may be expressed as $F = \frac{V_0 - v}{V_0}$.

It is assumed that after each administration of blood, sufficient time elapses to insure perfect mixing before any subsequent withdrawal. If an injection of blood precedes the withdrawal, then the above equation becomes $F = \frac{V_0}{V_0 + v}$. After $n$ successive exchanges, the percentage of the patient’s whole blood remaining in the body is equal to $100 F^n$. 
The above formulae apply to the red cells as well as to the whole blood if the ratio of donor to recipient hematocrit \( R \) is unity.

For \( R \neq 1 \), \( F \) is the same but the dilution of the red cells depends also on the initial and final hematocrits. The final hematocrit can only be determined by calculating the ratio of the hematocrits at each successive step.

Let \( H_0 \) = initial hematocrit of the patient,
\( H_1 \) = hematocrit of the patient after the first discrete exchange,
\( H_n \) = hematocrit of the patient after \( n \) exchanges,
\( H_d \) = hematocrit of the blood used for exchange. Then

\[
R = \frac{H_0}{H_1}, \quad R_1 = \frac{H_1}{H_1}, \quad \ldots, \quad R_n = \frac{H_n}{H_n}.
\]

Then, after the first discrete exchange, the patient's hematocrit becomes

\[
H_1 = \frac{100}{V_0} \left( \frac{V_0 - v}{V_0} \frac{1}{R} + \frac{v}{V_0} \right)
\]

and

\[
H_n = \frac{100}{V_0} \left( \frac{V_0 - v}{V_0} \frac{1}{R_{n-1}} + \frac{v}{V_0} \right)
\]

for \( n = 1, 2, 3, \ldots \).

Let \( C_E^R \) be the per cent concentration of the original red cells after a total volume exchange of \( E \) times \( V_0 \) and for an initial hematocrit ratio \( R \). Then,

\[
C_E^R = \frac{\text{volume of original red cells remaining}}{\text{final total red cell volume}} = \frac{V_0 F^n \left( \frac{H_0}{100} \right)}{V_0 \left( \frac{H_n}{100} \right)} = 100 \left( \frac{R_n}{R} \right)
\]

For \( n \) sufficiently large, \( R_n \) approaches unity and the above formula becomes:

\[
C_E^R = 100 \left( \frac{F^n}{R} \right)
\]

Thus, if \( v = \frac{V_0}{10} \), \( R_0 \) (i.e. hematocrit ratio after 10 discrete exchanges or after a total exchange of \( E = 2V_0 \)) becomes 1.07 for the initial hematocrit ratio \( R = 2 \), 1.10 for \( R = 4 \) and 0.89 for \( R = 1/2 \). The corresponding values of \( R_0 \) (i.e. for \( E = 4V_0 \)) are 1.01, 1.01 and .985 respectively, and may be replaced by \( v \) in the equation for \( C_E^R \) without introducing an error of more than \( 1 \) to \( 2 \) per cent. Similar expressions may be derived for any particular variation of the discrete transfusion.

Curves drawn from the above data are seen in figure 1 in which the efficiency of a continuous volume-for-volume substitution transfusion is compared with variations of the discrete type. The per cent dilution of original cells remaining in the circulation, \( C_E^R \), is determined for \( R = 1 \), i.e., for the case when the donor and patient hematocrits are the same. As in discrete exchange transfusions, \( E \) is the volume of donor blood used expressed in multiples of the patient's initial blood volume. The topmost curve (1) is for a continuous volume-for-volume exchange.
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with the circulating blood volume remaining unchanged during the whole procedure. (The equation for this curve is derived in the next section.) The next three

![Graph showing comparison of continuous and discrete exchanges for the case of donor to patient hematocrit ratio, R = 1.](image)

**Fig. 1.**—Comparison of continuous and discrete exchanges for the case of donor to patient hematocrit ratio, \( R = 1 \). (1) Continuous exchange. (2) Discrete exchange, \( V_0/20 \) removed and added alternately. (3) Discrete exchange, \( V_0/10 \) removed and added alternately. (4) Discrete exchange, \( V_0/5 \) removed and added alternately. Original cells remaining in the circulation at any time during or at the end of an exchange transfusion in per cent of total circulating cells \( (C_p) \) is plotted against the ratio of the total volume of donor blood used to original blood volume of the patient \( (E) \).

**Example:** Suppose that we want to perform a replacement transfusion using a total volume equal to three times the patient's original blood volume, i.e., \( E = 3.0 \). If the donor-patient hematocrit ratio is unity, we can achieve one of the following results:

1. Continuous exchange: \( C_p = 5\% \).
2. Discrete exchange, with \( r = V_0/20 \) removed and added alternately: \( C_p = 4.7\% \).
3. Discrete exchange, with \( r = V_0/10 \) removed and added alternately: \( C_p = 4.3\% \).
4. Discrete exchange, with \( r = V_0/5 \) removed and added alternately: \( C_p = 3.5\% \).

In the case of a discrete exchange with a volume \( r \) first administered and then withdrawn, the \( C_p \) will be greater than in a continuous exchange, and will increase with increasing \( r \). The exact values can be easily obtained by using the formula \( C_p = 100 \left( \frac{V_0}{V_0 + r} \right)^n \).

curves demonstrate the efficiency of discrete exchanges with one-twentieth, one-tenth and one-fifth of the patient's initial blood volume removed and injected alter-
As can be seen, the discrete method with the initial removal is slightly more efficient in exchanging red cells than the continuous one, and the greater the increments of blood \( (r) \) used, the better the replacement. However, it is difficult to remove more than one-tenth of the patient’s blood volume without some risk to the patient. At \( E \) equal to 1.5, there is 2 per cent more efficient replacement in the one-tenth volume discrete method than in the continuous one. At two volumes this difference is 1.5 per cent and at 4 volumes there is less than 0.5 per cent difference (initial red cell concentration is taken as 100 per cent). In adults we favor the continuous over the discrete method since it is easier, faster and less shocking to the patient; the circulating blood volume remains unchanged, and the difference in efficiency in favor of the discrete method is slight. The discrete method using the umbilical vein is the most feasible in infants.

**Figure 2.** Discrete exchange transfusions with \( V_1/10 \) removed and added alternately, for \( R = 0.5, 1, 2 \) and 4. The curves show the effect of increasing the donor-patient hematocrit ratio \( (R) \) on the per cent concentration of original cells in the total red cell volume after equal volume exchanges.

*Example:* If a discrete replacement transfusion with \( E = 2.5 \) and \( r = V_1/10 \) is performed we obtain the following \( C_{rd} \) for various donor-patient hematocrit ratios, \( R \):

1. For \( R = 0.5, C_{rd} = 15\% \)
2. For \( R = 1.0, C_{rd} = 7.1\% \)
3. For \( R = 1.0, C_{rd} = 3.7\% \)
4. For \( R = 4.0, C_{rd} = 1.9\% \)
volume exchange, $C^2_0$ is 35 per cent at $R = 1$, 21 per cent at $R = 2$ and 12 per cent at $R = 4$. For a two-volume exchange, $C^2_0$ at $R = 2$ and $R = 4$ is approximately $\frac{1}{2}$ and $\frac{1}{4}$ respectively the value at $R = 1$.

**Continuous Exchange**

Let $V_0$ = initial blood volume of the patient,
$V_1$ = blood volume removed prior to, and added at the end of, the continuous exchange,
$V$ = $V_0 - V_1$ = blood volume of the patient during the continuous exchange,
$V_r$ = instantaneous total red cell volume in circulation during the continuous exchange,
$T$ = volume of donor's blood used during the exchange,
$E = \frac{V}{V_0} - V_1$, i.e., the volume of donor's blood used, expressed in multiples of $V$ (the volume exchange),
$H_d$ = hematocrit of the blood used for the exchange,
$H_0$ = initial hematocrit of the patient,
$R = \frac{H_d}{H_0}$,
$P$ = the instantaneous volume of patient's original red cells remaining in the circulation during the continuous exchange.

In the above, $V_r$, $T$, $E$ and $P$ are variables; the remaining symbols represent constants of the exchange. We assume that during the continuous exchange, transfusion blood is administered to and withdrawn from the patient simultaneously, and at the same rate, and that the mixing of the circulating blood is so rapid that it may be considered instantaneous. At the beginning of the continuous exchange, $E = 0$, and $P = \left(\frac{H_0}{100}\right)V$. The volume of the patient's original red cells removed with each withdrawal of a small volume of patient's whole blood is equal to that volume of the whole blood removed multiplied by the ratio $\frac{P}{V}$:

$$dP = -\frac{P}{V} dT$$

(Where the minus sign indicates that $P$ decreases as $T$ increases).

Since $T = VE$ and $V$ is a constant during the continuous exchange,

$$dP = -\frac{P}{V} d(VE) = -\frac{P}{V} V dE = -P dE$$

$$dP = -dE.$$ Integrating this equation,

$$\int \frac{dP}{P} = -\int dE \text{ or, } \ln P = -E + \ln C.$$ Here $\ln C$ stands for a constant of integration to be determined;

hence $\ln \frac{P}{C} = -E$, or $\frac{P}{C} = e^{-E}$, and solving for $P$:

$$P = Ce^{-E}. \quad (i)$$

When $E = 0$, $P = \left(\frac{H_0}{100}\right)V$ and $\left(\frac{H_0}{100}\right)V = C \Rightarrow C$. Substituting this value of $C$ into $(i)$ we get:

$$P = \left(\frac{H_0}{100}\right)Ve^{-E} \quad \text{Formula 1.}$$

The change in patient's instantaneous total red cell volume while a small volume of blood is administered and withdrawn (simultaneously) is equal to the volume
administered multiplied by $(H_{d}/100)$ minus the volume withdrawn multiplied by $V_r$:

$$dV_r = (H_{d}/100)\ dT - \frac{V_r}{V}\ dT.$$  

Since $T = VE$ and $V$ is a constant, this becomes:

$$dV_r = \left(\frac{H_{d}}{100} - \frac{V_r}{V}\right) d(VE) = \left(\frac{H_{d}}{100}V - V_r\right) dE.$$  

Separating the variables and integrating we get:

$$\int \frac{dV_r}{(H_{d}/100)V - V_r} = \int dE,$$

hence

$$-\ln \left[(H_{d}/100)V - V_r\right] = E - \ln C,$$  

and

$$(H_{d}/100)V - V_r = Ce^{E}.$$  

(2)

When $E = 0$, $V_r = (H_{d}/100)V$, and therefore Equation (2) becomes

$$(H_{d}/100)V - (H_{0}/100)V = (H_{0}/100)V(R - 1);$$

substituting this value of $C$ into (2) we get:

$$V_r = (H_{d}/100)V(1 - R) = (H_{0}/100)\ V[1 - (R - 1)\ e^{-E}];$$

or

$$V_r = (H_{0}/100)\ V (R - (R - 1)\ e^{-E}).$$  

Formula 2.

Since $C_0^R = \frac{Volume\ original\ red\ cells\ remaining}{Total\ red\ cell\ volume} = \frac{P}{V_r}$, we can now express it in terms of $V_r$ and $E$:

Formulae 1 and 2:

$$C_0^R = \frac{P}{V_r} = \frac{(H_{d}/100)Ve^{-E}}{(H_{0}/100)V(1 - (R - 1)e^{-E})};$$  

or simply,

$$C_0^R = \frac{e^{-E}}{R - (R - 1)e^{-E}} = \frac{1}{R(e^{-1}) + 1}.$$  

Formula 3.

Curves drawn from Formula 3 for different values of $R$ are given in figure 3. The per cent concentration of original cells remaining in the circulation $(C_0^R)$ is plotted against volume exchange $(E)$. It is apparent that as $R$ increases, greater dilution of the patient's erythrocytes occurs and a more effective exchange is produced with the same quantity of blood. There is a point beyond which the process of diminishing returns would render further exchange wasteful, time consuming, and of questionable benefit. For $R = 2$, a $C_0^R$ of 10 per cent can be accomplished with less than two volumes exchange, whereas a $C_0^R$ of 5 per cent will require about three volumes replacement. It is evident that the ratio of donor to patient hematocrit is extremely important in both the discrete and continuous exchange transfusions in achieving the desired $C_0^R$ value.

Figure 3 may well serve as a master chart for those interested in determining the pertinent data for a replacement transfusion.

Numerous variations of the continuous exchange may be employed; a known volume of blood may be removed or added before the continuous exchange is
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FIG. 3.—Theoretic curves for continuous exchange transfusions for different values of R. Note effect of increasing the ratio of the donor to the patient hematocrit (R) on per cent concentration of original cells (C_R^o).

Example: Suppose that we want to determine the amount of donor's blood, T, that is required for a continuous replacement transfusion in order to dilute the patient's original erythrocytes twenty times, i.e., to obtain a C_R^o of 5 per cent. Let us assume that the patient's total blood volume is V = 5500 cc., the hematocrit is 2.5 per cent, and that the donor hematocrit is 40 per cent. Then the donor-patient hematocrit ratio is R = 1.6. From figure 3 we see that C_R^o = 100 = 5 per cent; hence by interpolating we find that for R = 1.6 the required E = 2.56; therefore, the total volume to be used in the exchange transfusion is T = E x V = 2.56 x 5500 = 14,080 cc.

started, and the blood volume may be brought to the original level or a higher or lower level during or at the end of the exchange. For each of the above combinations an appropriate mathematical expression may be derived.

For the case where a known volume V_1 is removed prior to, and replaced at the end of, the exchange, let C_R^o, V_t stand for the per cent concentration of the original red cells remaining in the body at the end of the transfusion (i.e., after the final addition of V_1):

\[ C_R^o = \frac{P}{V_t} \]

Where P and V_t are defined as:

- \( P = \text{volume of the original cells remaining at the end} \)
- \( V_t = \text{total volume of the red cells at the end} \)

It is desirable to express \( C_R^o \) in terms of \( C_R^o \) and to reduce all our calculations to simple arithmetic operations performed on data which can be read off the master chart of figure 3. To this end we proceed as follows: applying Formula 1 and noting the fact that for \( R = 1 \), Formula 3 reduces to \( C_R^o = 100 \cdot R \), we get:

\[ P = \frac{(H_0/100)V_t}{100} = \frac{V_t}{100} \cdot \frac{VC_R^o}{100} \]
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To obtain the required expression for $V_r$, we start with the definition of $C_E^R$:

$$C_E^R = \frac{100}{V_r},$$

and therefore

$$V_r = \frac{100}{C_E^R} = \frac{100}{P} + \frac{(H_d/100)\nu_1}{C_E^R}$$

Inserting (j) and (k) into the definition of $C_E^{R+V_1}$, we get

$$C_E^{R+V_1} = \frac{100 \nu_v}{V_v} = \frac{100 \nu_v}{\nu_v + (H_d/100)\nu_1} = \frac{100PV_E^{R+V_1}}{100PV_E^{R+V_1} + (H_d/100)\nu_1}$$

which finally reduces to

$$C_E^{R+V_1} = \frac{VC_E^R \nu_1 C_E^R}{VC_E^R + RV_1 C_E^R}$$

Formula 4

Note that when $\nu_1 = 0$, this reduces, as required, to $C_E^{R+V_1} = C_E^R$, and that for the special case of $R = 1$ the equation becomes:

$$C_E^{1+V_1} = \left( \frac{V}{V + \nu_1} \right) C_E^1 = \frac{V_0 - \nu_1}{V_0} C_E^1$$

The effect on a continuous exchange of an initial removal of one-fifth of the blood volume for the cases $R = 1$ and $R = 4$ is seen in figure 4. In this figure, the scale $E_0$ refers to the administered blood volume expressed in multiples of the original total blood volume $V_0$, while $E$ refers to the volume expressed in multiples of $V$, i.e., of the circulating total volume after the initial removal of $V_1 = \frac{1}{5}V_0$. The circulating volume to be exchanged after removing $\frac{1}{5}V_0$ is only $\frac{1}{5}$ of the original volume and hence the same volume of blood used as the exchanging medium will be more effective than without the initial removal. In the topmost series of curves (a, b, c) for $R = 1$, (a) represents a continuous exchange curve and (b) depicts the results when the circulating blood volume is first reduced by $\frac{1}{5}$ and then a continuous exchange is performed without replacing the removed blood at the end. As is seen, when a one-volume replacement ($E_0 = 1$) is performed, the initial removal of $\frac{1}{5}$ of the volume has no advantage over the pure continuous method. At 2-volumes replacement, however, there is a 1.5 per cent advantage in the case of the initial removal, and this advantage increases as $E$ increases. If, at the end of the exchange, the volume initially removed is replaced, additional dilution of the remaining erythrocytes occurs and $C_E^R$ is further decreased (c). Thus, after 2-volume exchange, the pure continuous method will leave about 13 per cent of the original cells, whereas if $\frac{1}{5}$ volume is removed initially and a similar volume added at the end, about 7 per cent of the original cells are in the circulation. Note that this advantage in dilution obtained by adding a volume at the end of an exchange holds for any volume replacement.

If the ratio of the hematocrit of the donor blood to the patient’s blood is increased, greater dilution of the patient’s remaining red cells occurs. Although the
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**Fig. 4.**—The effect on a continuous exchange (a, a') of an initial removal of $V_1 = V_o/5$ (b, b') and of the addition of an equal volume of donor blood at the end of the replacement transfusion (c, c'). Curves a, b, c are plotted for the case $K = 1$; curves a', b', c' for $K = 4$. With reference to curves c and c', the $C_e$ stands for $C_{e-V_1}$; in the case of curves b and b', $C_e$ denotes the per cent concentration of original erythrocytes after the initial withdrawal of $V_1$ but prior to the replacement of $V_1$. In the upper scale, $E = E_o$ refers to the blood volume withdrawn or administered in multiples of the original blood volume, $V_o$; in the lower scale $E$ refers to the volume of blood withdrawn or administered in multiples of $V_o$, the circulating blood volume during the exchange. Note that for curves b and b', $T$ stands for total volume withdrawn only, and is equal to the total volume administered plus $V_1$.

**Example.** Suppose that in performing a replacement transfusion we want to achieve a $C_e$ of 2 per cent. Let us further assume that the patient's original total blood volume is $V_o = 5500$, that his hematocrit is 12 per cent and that the hematocrit of the donor blood is 48 per cent. The donor-patient hematocrit ratio is then $K = 48/12 = 4$.

Case 1: From figure 4, considering curve a' and using the upper scale for $E_o = T/V_o$, we see that for a continuous replacement transfusion (without any initial withdrawal) the required $E_o = T/V_o = 2.58$, and hence $T = E_o \times V_o = 2.58 \times 5500 = 14,300$ cc.

Case 2: If, however, we withdraw 2 per cent of the patient's initial blood volume (i.e., $V_1 = 1100$ cc.) prior to the continuous replacement transfusion and administer it again at the end, then the $E_o$ (upper scale) corresponding to $C_{e-V_1} = 2$ per cent is given on the curve c' as 2.08 and hence the total donor blood volume required is $T = E_o \times V_o = 2.08 \times 5500 = 11,440$ cc., i.e., 1,760 cc. less than in Case 1. Note that only 10,340 cc. are used during the continuous replacement transfusion. The calculations implicit in Case 2 can be broken into their component parts, namely, into that for a continuous exchange transfusion, and for a single discrete transfusion. After the initial removal of 1,100 cc., the patient's blood volume during the continuous exchange is $V = 4400$ cc., $R = 4$, as before and $E = 0.4V$ of donor's blood used during the continuous exchange divided by $V_o = 10,340/4,400 = 2.35$. This corresponds to a $C_{e-V} = 2.35$ per cent as can be seen either from curve a' using the upper scale $E_o$, or from curve b' using the lower scale $E$ (curves b and b' begin with 0.25 on the lower scale instead of zero; hence the reading is taken at $E = 2.35 + 0.25 = 2.60$). The corresponding hematocrit can be read off the chart as described in the text and is equal to 44 per cent. Since the patient's original red cells are further diluted by a final addition of 1,100 cc. of donor blood (hematocrit 48 per cent), the final per cent concentration of original erythrocytes becomes 2.35 (4400 $\times$ 44 / (4400 $\times$ 44 + 1100 $\times$ 48)) = 2.35 0.78 = 1.70 per cent as required.
total volume of the original erythrocytes removed (efficiency) is the same for any value of \( R \), the per cent concentration of original cells in the circulation is decreased as \( R \) increases. As is seen in figure 4 there is a marked decrease in \( C^E \) with increase in \( R \) for the same volume exchanged. In the case of a 2-volume exchange, for \( R = 1 \) about 7 to 13 per cent of the patient's cells remain in the circulating blood (a, b, c); for \( R = 4 \), 4 to 6 per cent remain (a', b', c'). For intermediate values of \( R \), intermediate values for \( C^E \) are obtained.

The circulating hematocrit at the end of (or during) a continuous exchange can also be determined from these formulae:

Let \( H_p \) = the circulating hematocrit at any time; then, using Formula 2 to express the value of \( V_r \),

\[
H_p = 100 \left[ \frac{V_r}{V} \right] = 100 \left[ \frac{(H_0/100)V(R - (R - 1)e^{-R})}{V} \right] = H_0[R - (R - 1)e^{-R}];
\]

multiplying and dividing by \( 100e^{-R} \) we can write this as

\[
H_p = H_0 \left[ \frac{R + (R - 1)e^{-R}}{100e^{-R}} \right] \approx \frac{H_0}{100} C^E \]

which, via Formula 1, is equivalent to:

\[
H_p = H_0 \frac{C^E}{100}
\]

Since the above relationship exists between each of the continuous exchange curves for various values of \( R \) and the curve for \( R = 1 \), figure 3 may be used for determining the circulating hematocrit at any time during or at the end of an exchange transfusion. A logarithmic ruler or scale is placed vertically at the desired value of \( E \). The ruler is moved so that the value for the initial hematocrit read on the scale coincides with the \( R \) curve applicable to the case; with the ruler in this position the final circulating hematocrit may be obtained by reading the value on the ruler at \( R = 1 \). Thus, if \( H_0 = 20 \) per cent, \( R = 2 \), and \( E = 2 \), the circulating hematocrit \( H_p \) will equal 37 per cent.

From the above information it is possible to determine the conditions for an optimum exchange transfusion in a particular case. Since, by definition,

\[
C^E = \frac{\text{volume of original red cells remaining}}{\text{total volume of red cells remaining}}
\]

it is in general desirable to make the numerator, or the volume of original cells remaining, as small as possible, and the denominator as large as feasible.

The number of original red cells remaining is a function of the whole blood volume used in the exchange and can be improved only by reducing the patient's blood volume prior to the exchange and keeping it at that level during the exchange, before bringing it back to the desired level at the end (fig. 4).

The denominator on the other hand is completely independent of the actual exchange procedure and depends only upon the final hematocrit of the patient's blood. The end result will thus be the same whether we start by injecting low hematocrit blood and finish the exchange with packed red cells or vice versa, as long as the patient's final hematocrit is the same. The first procedure might be
more economical since a smaller red cell volume is used. This is demonstrated in
the following example:

Suppose that the patient's initial hematocrit is 20 per cent and that the mini-
mum requirements for an exchange transfusion are: (a) $C_r^k$, the final concentra-
tion of the original red cells to be not more than 5 per cent (i.e., a dilution of 95 per
cent); (b) the final hematocrit of the patient to be not less than 36 per cent. We
can proceed in one of several ways to solve this problem:

1. By definition, $C_r^k = \frac{P}{V_r}$ where $P = \frac{H_0}{100} V e^{-k}$ (Formula 1) or since
$100e^{-k} = C_r^k$ (Formula 3), $P = \frac{H_0}{100} V (C_r^k / 100)$. On the other hand $V_r$ (the
volume of red cells at the end of the continuous exchange) can be expressed as
the patient's final hematocrit divided by 100 and multiplied by the total blood
volume, $V_r = (H_f/100)V_0$. Substituting the desired values into these equations
we get:

$$ C_r^k = \frac{(H_0/100)V(C_r^k/100)}{(H_f/100)V} = \frac{H_0 C_r^k}{H_f} \text{ or} $$

$$ 5\% = \frac{0.25C_r^k}{36} \text{ and hence } C_r^k = 9\%. $$

From figure 3 we see that $C_r^k = 9$ per cent when $E = 2.4V_0$. Let us further assume
that the maximum permissible hematocrit of the blood used for transfusion is
60 per cent; then, using the logarithmic ruler in a way similar to that described
above, we find that for a hematocrit ratio of 3 (60 per cent

20 per cent), an exchange of

$E = 0.5V_0$ will raise an initial hematocrit of 20 per cent to 36 per cent. Hence
the desired result will be achieved by performing first an exchange of $E = 1.9V_0$
with donor blood having an hematocrit of 20 per cent ($R = 1$), and by following
this with an exchange of $E = 0.5V_0$ using blood with an hematocrit of 60 per
cent ($R = 3$). The total red cell volume used in this exchange is $20/100) 1.9V_0 +
(60/100)0.5V_0 = 0.68V_0$.

The same result can be achieved by first performing an exchange of $E = 0.5V_0$
with donor blood having an hematocrit of 60 per cent ($R = 3$), and by following
this with an exchange of $1.9V_0$ using blood with an hematocrit of 36 per cent
($R = 1$). The total red cell volume used in this exchange is $60/100) 0.5V_0 +
(36/100)1.9V_0 = 1.04V_0$.

2. If we administer blood with hematocrit of 40 per cent (i.e., $R = 2$) we
achieve the required $C_r^k$ of 5 per cent and a final hematocrit of about 37 per
cent when $E = 2.35V_0$. The corresponding red cell volume used is $(40/100)2.35V_0 =
0.94V_0$.

3. Finally, if we only use blood with 60 per cent hematocrit ($R = 3$), we
get a $C_r^k$ of 5 per cent when $E = 2.0V_0$, but the final hematocrit is now 34 per
cent. The corresponding red cell volume used is $(60/100)2.0 = 1.2V_0$.

In the case where the removal of other blood constituents is important as in
uremia due to anuria, all the above calculations are still applicable. However,
R will now stand for the ratio of the percentages of the particular constituents in the donor and patient whole blood. For instance, in uremia with a urea nitrogen of 150 mg. per cent, R will be \( \frac{1}{10} \) assuming that the donor blood has 15 mg. per cent of urea nitrogen. The data for the curve \( R = \frac{1}{10} \) can be easily calculated from the fundamental Formula 3. This method of determining the extent of a replacement transfusion can be applied to changes in the concentration of other blood constituents.

**SUMMARY**

1. Theoretic effectiveness of different types of exchange transfusions has been determined.
2. Homogeneous and nonhomogeneous solutions used as the exchanging medium differ in final results obtained.
3. The final concentration of original erythrocytes (in per cent of total red cells in the circulation) diminishes as the ratio of donor to patient's initial hematocrit increases for the same volume exchange.
4. Final hematocrit values may be obtained from the calculated data.
5. Optimum considerations for exchange transfusions are discussed.

**REFERENCES**

BLOOD EXCHANGE IN REPLACEMENT TRANSFUSIONS: I. THEORETIC CONSIDERATIONS

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